Minimum Spanning Trees 1

Definition Recall that a spanning tree of a graph is a connected subgraph with no cycles that includes all the vertices. A minimum spanning tree (MST ) of an edge-weighted graph is a spanning tree whose weight (the sum of the weights of its edges) is no larger than the weight of any other spanning tree.

Assumptions

->The graphs is connected.

->The weights are not necessarily

->The edge weights may be zero or negative.

->The edge weights are all different.

## Underlying principles

Two basic properties of a tree:

->Adding an edge that connects two vertices in a tree creates a unique cycle.

->Removing an edge from a tree breaks it into two separate subtrees.

A black and red lines and dots

Description automatically generated

Cut property

Definition: A cut of a graph is a partition of its vertices into two nonempty disjoint sets. A crossing edge of a cut is an edge that connects a vertex in one set with a vertex in the other.

A diagram of a structure

Description automatically generated

Theorem. Cut Property. Given any cut in an edge-weighted graph, the crossing edge of minimum weight is in the MST of the graph.

Greedy algorithm

The following method colors black all edges in the the MST of any connected edge-weighted graph with V vertices: starting with all edges colored gray, find a cut with no black edges, color its minimum-weight edge black, and continue until V-1 edges have been colored black.

e.g.

A diagram of a network

Description automatically generated with medium confidence

## Edge-weighted graph data type

A screenshot of a computer code

Description automatically generated

A screenshot of a graph

Description automatically generated

A diagram of a graph

Description automatically generated